Applying Gauge Repeatability and Reproducibility Analysis for a Cast Dimension in a Foundry – A Case Study

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In the current manufacturing scenario, much emphasis is placed on identifying and reducing process variation. In the shopfloor, this is achieved through a systematic application of statistical process control techniques to various process parameters. In order that the manufacturing variation is quantified correctly, it is important that the instrument/equipment used to measure the process parameter/characteristic is assessed for its inherent capability. Although the capability assessment of a measurement system involves determination of both location and variability of the measurements generated by a measurement system, in this paper, the variability assessment part of the capability studies of a measuring instrument used in a statistical process control studies in a jobbing foundry has been assessed. The inferences regarding the suitability of the measurement system has been drawn on the basis of the guidelines proposed by the Automotive Industry Action Group (AIAG).

Introduction

Measurement data has numerous applications. To give an example, the decision on whether to make any adjustments to a manufacturing process or leave it alone is based on the data generated from the process. Statistical process control studies involve collection of measurement data. Statistical treatment of the measurement data helps in better understanding of the manufacturing process. It should however be kept in mind that the benefit obtained from a measurement data is high only if the quality of the data is also high.

Quality of measurement data is defined by the statistical properties of multiple measurements obtained from a measurement system operating under stable conditions. The statistical properties most commonly used to characterise the quality of the data are the Bias and the Variance of the measurement system. Bias refers to the location of the data relative to the master value. Variance refers to the spread of the data. One of the common reasons cited for low quality data is large variance.

Measurement System

Measurement system is a collection of instruments or gauges, standards, operations, methods, fixtures, softwares, personnel, environment and assumptions used to quantify a unit of measure or fix assessment to the feature characteristic being measured; the complete process used to obtain measurement [1].

A measurement system can also be viewed as a process as shown in Fig. 1.

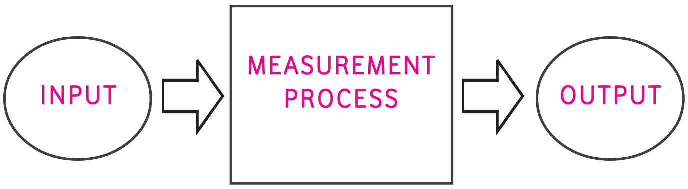


Fig. 1: Viewing Measurement system as a Process.

Measurement Issues

Some of the pertinent questions related to a measurement system are:

- Does the measurement system have adequate discrimination/ resolution?
- Is the measurement system statistically stable over time?
- Is the measurement variation small?

In this paper, concern is about the third question wherein the variation associated with a measurement system is assessed and compared with the standard guidelines for acceptance established by AIAG.

Measurement System Errors

Measurement system errors are classified into five categories. They are :

- Bias
- Linearity
- Stability
- Repeatability
- Reproducibility

Out of these five, bias, linearity and stability concern themselves with the location of the measurement data and repeatability and reproducibility concern themselves with the spread of the measurement data.

Components of Measurement Variation

Since the measurement is thought of as a process, the various components of the measurement process and their interactions contribute to the variation in the measurement data. Figure 2 illustrates the various contributors to the measurement error or the variation in the measured values.

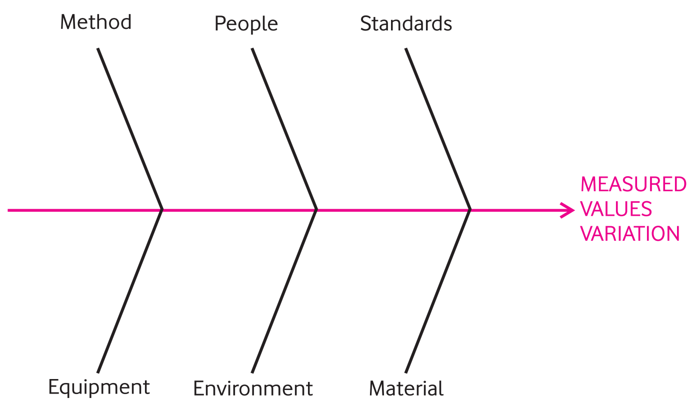


Fig. 2: Contributors to the Measurement Errors.

In general, the data collected for controlling a manufacturing process contains variations from two different and independent sources.

- Manufacturing process variation
- Measurement system variation.

The above-mentioned components of variation put together are called Total variation. Figure 3 illustrates the above-mentioned sources of variation graphically and also how these 2 sources of variation get added up statistically to give the total variability.

Figure 4 illustrates the components of a measurement system analysis. As shown in Fig. 4, the total variation observed in a manufacturing process can be attributed to 3 sources namely,

- Time-to-Time Variation.
- Part-to-Part Variation.
- Within-Part Variation.

The measurement system variation is classified into 2 categories namely,

- Instrument variation.
- Measurement process variation.

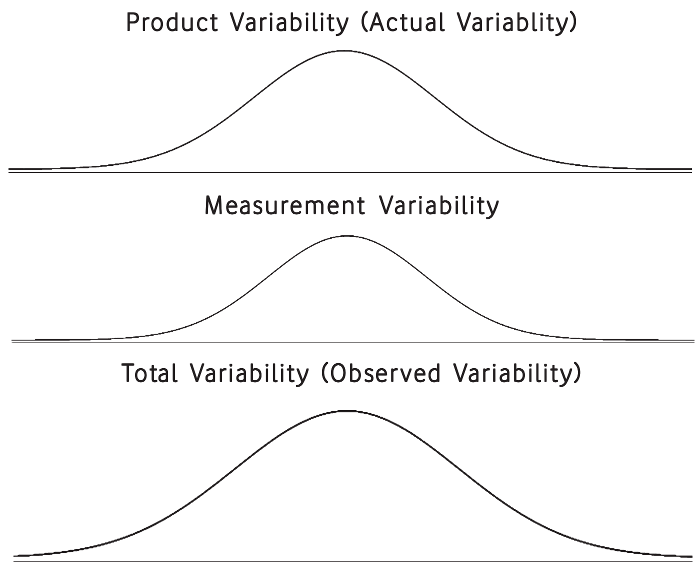


Fig. 3: Sources of variation.

The instrument variation can be due to,

- Bias.
- Linearity.
- Stability.
- Repeatability.

Measurement process variation is due to poor reproducibility. The aspects of the measurement system analysis that are focused in this paper are enclosed within dotted boxes.

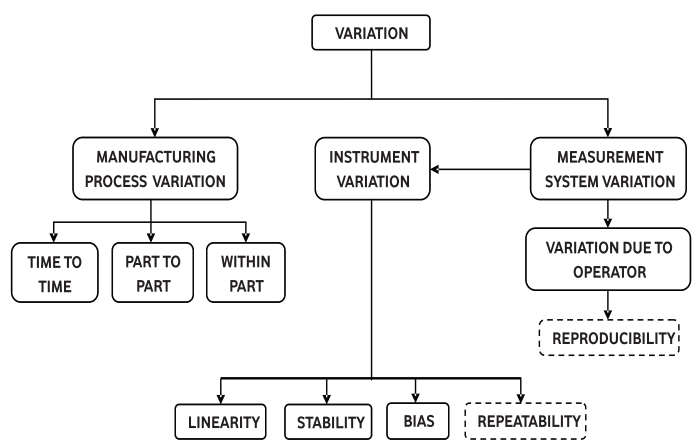


Fig. 4 : Components of Measurement System Analysis.

Total Variation of a Manufacturing Process

In order that a measurement process accurately captures the process variation, it is extremely important that the measurement variation must be smaller than the corresponding manufacturing process variation. Otherwise, the measurement system variation which by itself may be small thus gives a false indication of high process variability.

In general,

$\mu_{total} = \mu_{product} + \mu_{measurement}$

$\sigma^2_{total} = \sigma^2_{product} + \sigma^2_{measurement}$

Where,

$\mu_{product}$ = product average

$\mu_{measurement}$ = measurement system bias

$\sigma^2_{product}$ = product variance

$\sigma^2_{measurement}$ = measurement system variability

Measurement system bias is determined through the study of bias, linearity and stability of a measurement system. Measurement system variability is determined through Gauge Repeatability and Reproducibility studies.

Gauge Repeatability and Reproducibility Study

Repeatability and reproducibility deals with the width or spread of the measurement system variation.

Repeatability (σ_G)

It is the variation in the measurements obtained with one measurement instrument when used several times by one appraiser while measuring the same characteristic on the same part.

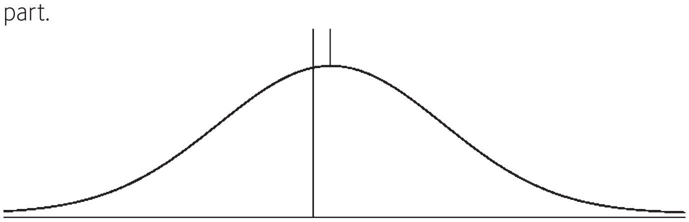


Fig. 5: Repeatability.

Repeatability is the **inherent variability** or capability of the equipment itself.

Reproducibility (σ_A)

It is the variation in the average of measurements made by different appraisers using same measuring instrument when measuring the same characteristic on the same part. It is the **appraiser variability** of the measurement system.

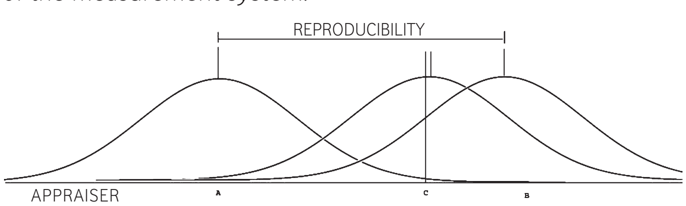


Fig. 6: Reproducibility.

Procedure Followed for Conducting GR&R (Gauge R&R) Study

- The instrument used for GRR study in the identified foundry was a vernier calliper.
- 3 appraisers who are users of the measurement systems were selected.
- 10 components that represent the expected range of process variations were used in the study.
- The 10 components were numbered from 1 to 10 in such a way that these numbers were not visible to the appraisers.
- The 10 components were measured in random order by appraiser A with an observer recording the results.
- The above step was repeated with other appraisers while at the same time ensuring statistical independence. This means that the data are collected in a random manner so as to avoid the bias in the values. A simple way to assure a balanced design for (10) parts, (3) appraisers, and (3) trials is through randomisation [Page 117, reference 1]. The readings of the various appraisers were concealed in order to avoid any bias.
- The above 2 steps were repeated using a different random order of measurement.

Data Collection for GRR Study

Table-1 shows the format standardised by AIAG for collecting the data pertaining to GR&R study [Page 216, reference 1]. The GRR data table that could be used for further analysis is shown in Table-2.

Table -1 : Standard Data Collection Form

Gauge Repeatability and Reproducibility Data Collection Sheet											
Appraiser /Trial#	PART										AVERAGE
	1	2	3	4	5	6	7	8	9	10	
A 1											
2											
3											
Average											$\bar{X}_a$ =
Range											$\bar{R}_a$ =
B 1											
2											
3											
Average											$\bar{X}_b$ =
Range											$\bar{R}_b$ =
C 1											
2											
3											
Average											$\bar{X}_c$ =
Range											$\bar{R}_c$ =
Part Average											$\bar{\bar{X}}$ = $\bar{R}_p$ =
([$\bar{R}_a$ =] + [$\bar{R}_b$ =]+[$\bar{R}_c$ = D/[#OF APPRAISERS =] =											$\bar{\bar{R}}$ =
$\bar{X}_{avr}$ = [Max $\bar{X}$ =] – Min $\bar{X}$=] =											$\bar{X}_{DIFF}$ =

Analysis Based on Average and Range Method

Average and Range method is an approach which will provide an estimate of both repeatability and reproducibility.

Table-2: GRR Data Sheet.

Gauge Repeatability and Reproducibility Data Collection Sheet

Appraiser /Trial #	PART 1	2	3	4	5	6	7	8	9	10	AVERAGE	
A 1	30.16	30.16	30.14	30.18	30.18	30.16	30.06	30.24	30.18	30.4		30.18
2	30.14	30.14	30.14	30.18	30.2	30.28	30.14	30.22	30.2	30.16		30.18
3	30.12	30.14	30.14	30.16	30.2	30.4	30.14	30.28	30.24	30.16		30.19
Average	30.14	30.146	30.14	30.173	30.193	30.28	30.113	30.246	30.206	30.24	$\bar{X}_a$ =	30.187
Range	0.04	0.02	0	0.02	0.02	0.24	0.08	0.06	0.06	0.24	$\bar{R}_a$ =	0.078
B 1	30.24	30.34	30.3	30.32	30.42	30.2	30.14	30.12	30.3	30.28		30.26
2	30.14	30.2	30.18	30.2	30.34	30.2	30.2	30.22	30.3	30.44		30.24
3	30.24	30.2	30.14	30.18	30.2	30.18	30.18	30.3	30.18	30.28		30.20
Average	30.206	30.246	30.206	30.233	30.32	30.193	30.173	30.213	30.26	30.333	$\bar{X}_b$ =	30.238
Range	0.1	0.14	0.16	0.14	0.22	0.02	0.06	0.18	0.12	0.16	$\bar{R}_b$ =	0.13
C 1	30.34	30.2	30.22	30.24	30.18	30.2	30.2	30.18	30.34	30.18		30.22
2	30.12	30.26	30.1	30.22	30.3	30.32	30.18	30.2	30.32	30.32		30.23
3	30.2	30.2	30.1	30.2	30.14	30.22	30.2	30.22	30.16	30.3		30.19
Average	30.22	30.22	30.14	30.22	30.206	30.246	30.193	30.2	30.273	30.266	$\bar{X}_c$ =	30.218
Range	0.22	0.06	0.12	0.04	0.16	0.12	0.02	0.04	0.18	0.14	$\bar{R}_c$ =	0.11
Part Average	30.188	30.204	30.162	30.208	30.239	30.239	30.159	30.219	30.246	30.279	$\bar{\bar{X}}$ = R_p =	30.215 0.12

([$\bar{R}_a$ = 0.078] + [$\bar{R}_b$ = 0.13] + [$\bar{R}_c$ = 0.11]) / [# OF APPRAISERS = 3] = 0.106 | $\bar{\bar{R}}$ = 0.106

$\bar{X}_{diff}$ = [Max $\bar{X}$ = 30.238] - [Min $\bar{X}$ = 30.187] = 0.051 | $\bar{X}_{DIFF}$ = 0.050667

* UCL_R = [$\bar{\bar{R}}$ = 0106] × [D_4 = 2.58] = 0.273

*D_4 = 3.27 for 2 trials and 2.58 for 3 trials. UCL_R represents the limit of individual R's. Circle those that are beyond this limit. Identify the cause and correct. Repeat these readings using the same appraiser and unit as originally used or discard values and re-average and recompute $\bar{\bar{R}}$ and the limiting value from the remaining observations.

Notes:____________________________________

Range Chart

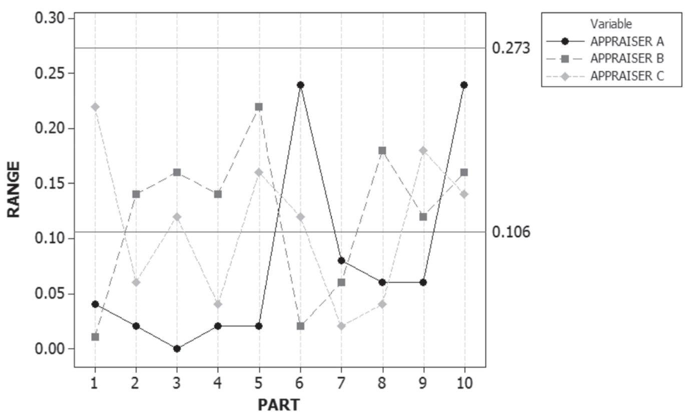
The range chart is used to determine whether the measurement process is in control. The ranges of the multiple readings by each appraiser on each part are plotted on a range chart that incorporates the average range and the control limits. If the ranges of all the appraisers are in control, it means that there is consistency in the measurement process between various appraisers for each part. If the range of one appraiser is out of control, it means that the method of measurement adopted by that appraiser is different from others. If the ranges of the multiple readings of all the appraisers are out of control, it means that either the measurement system is not suitable for measuring the given part dimension having the stated tolerance or is sensitive to the method of measurement adopted by each appraiser and needs improvement.

Figure 7 shows the range chart for the three appraisers A, B and C. Since the ranges of all the appraisers fall within the control limits,

Table-3: GRR Calculation Procedure

GAUGE REPEATABILITY AND REPRODUCIBILITY REPORT			
FROM DATA SHEET $\bar{\bar{R}}$ = 0.106 $\bar{X}$diff = 0.050667 $\bar{R}$p = 0.12			
MEASUREMENT UNIT ANALYSIS			% OF TOTAL VARIATION
Repeatability – Equipment Variation (EV) EV = $\bar{\bar{R}} \times K1$ = 0.106 x 0.5908 = 0.0626248	TRIALS	K1	%EV = 100(EV/TV) $= 100\left(\frac{0.0626248}{0.07693378549}\right)$ =81.40%
	2	0.8862	
	3	0.5908	
Reproducibility – Appraiser Variation (AV) AV * = $\sqrt{(Xdiff \times K2)^2 - \frac{0.0626248^2}{10 \times 3}}$ = $\sqrt{0.050667 \times 0.5231^2 - \frac{0.0626248^2}{10 \times 3}}$ = 0.02391084003 n = parts r = trials	APPRAISERS	K2	%AV = 100(AV/TV) $= 100\left(\frac{0.02391084003}{0.07693378549}\right)$ = 31.07%
	2	0.7071	
	3	0.5231	
Repeatability and Reproducibility (GRR) GRR = $\sqrt{EV^2 + AV^2}$ = $\sqrt{0.0626248^2 + 0.02391084003^2}$ = 0.06703427367	PART	K3	%GRR = 100(GRR/TV) $= 100\left(\frac{0.06703427367}{0.07693378549}\right)$ = 87.13%
	1	–	
	2	0.7071	
Part Variation (PV) PV = Rp x K3 = 0.12 x 0.3146 = 0.037752	3	0.5231	%PV = 100(PV/TV) $= 100\left(\frac{0.037752}{0.07693378549}\right)$ = 49.07%
	4	0.4467	
	5	0.4030	
	6	0.3742	
Total Variation (TV) TV = $\sqrt{GRR^2 + PV^2}$ = $\sqrt{0.067034273672^2 + 0.037752^2}$ = 0.07693378549	7	0.3534	ndc = $1.41\left(PV/GRR\right)$ $= 1.41\left(\frac{0.037752}{0.06703427367}\right)$ = 0.7940761805 = 1 (Pages 114, 217, reference 1)
	8	0.3375	
	9	0.3249	
	10	0.3146	

it is concluded that statistically, there is no difference in the method of measurement adopted by the three appraisers.



$UCL = D_4 \bar{\bar{R}} = 2.58 \times 0.106 = 0.273$

$LCL = D_3 \bar{\bar{R}} = 0 \times 0.106 = 0$

Fig. 7 : Range chart.

Average Chart

In this chart, the averages of the multiple readings by each appraiser on each part are plotted with part number as an index. This can assist in determining consistency between appraisers. The grand average and control limits determined by using the average range are also plotted. The resulting Average Chart provides an indication of "usability" of the measurement system.

The area within the control limits represents the measurement sensitivity ("noise"). In the present study, 10 different castings were used and these parts represent the variations inherent in the casting process. If the true performance of the casting process is to be obtained, then it is implicit that the variation in the measurement system used for assessing the casting process is either zero or minimum. This means that

- Process variation as revealed by the part-to-part variation is not masked or concealed by the measurement system variation.
- The measurement system used has adequate discrimination to detect part-to-part variation and thus indicate the true performance of the process.

This is facilitated if approx. one half or more of the averages fall outside the control limits. If the data show this pattern, then the measurement system should be adequate to detect part-to-part variation and the measurement system can provide useful information for analysing and controlling the process. If less than half of the plotted points fall outside the control limits, then either the measurement system lacks adequate effective resolution or the sample does not represent the expected process variation [Page 102, reference 1]. Figure 8 shows the average chart for the same three appraisers. Based on the discussion above, it can be concluded that the measurement system is not adequate enough to detect part-to-part variation as only 2 out of 10 points fall outside the control limits.

Analytical Method for Calculating GRR

The formulae used for calculating appraiser, part and total variation as well as repeatability and reproducibility along with the calculations are shown in Table-3 [Page 217, reference 1].

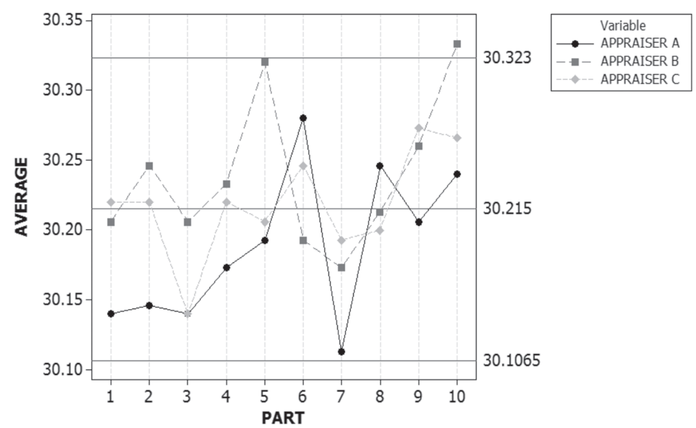


Fig. 8 : Average Chart.

$UCL = \bar{\bar{X}} + A_2 \bar{\bar{R}} = 30.215 + 1.023 \times 0.106 = 30.323$

$UCL = \bar{\bar{X}} - A_2 \bar{\bar{R}} = 30.215 - 1.023 \times 0.106 = 30.1065$

Acceptance Criteria Based on GRR (Proposed by AIAG)

S.No	GRR	DISPOSITION
1	< 10%	The measurement system is acceptable
2	10% < GRR < 30%	The measurement system may be acceptable based on application , cost of gauge, cost of repair etc.
3	>30%	The measurement system needs improvement or corrective action.

Analysis and Interpretation

- Equipment variation is found to be 81.4% which means that the equipment has to be replaced with a new one or some improvement action for reducing the equipment variation has to be initiated. Until such a time, this equipment is NOT TO BE USED either for SPC studies for assessing the process variation or for any other measurement purpose.
- Appraiser variation is found to be 31%. Although the range chart method of analysis indicated no significant difference in the method of measurement among the appraisers, the method of measurement adopted by the appraisers can still be studied to track down to any possible reasons of variation.

- Part variation is found to be 54%. The value of part-to-part variation seen in conjunction with the equipment variation sheds light on the fact that a portion of the part-to-part variation is masked by the equipment variation. This throws light on the necessity to take corrective actions, improve the equipment/ measurement system and also a relook on the process.
- Gauge repeatability and reproducibility is found to be 87%. This is because of high equipment and appraiser variation. Such a high value of GR&R indicates that the measuring equipment has to be discarded or improved and that it should not be used as such anymore for any measuring application.
- The number of distinct categories (ndc) is found to be 1 which is unacceptable as its value must be greater than or equal to 5.

Conclusion

In the present work, the method of analysing the variability of a measurement system was demonstrated. Such an assessment is extremely important before a measurement system is used in statistical process control studies. Such an approach would help better understanding and monitoring of the process variation which when reduced or eliminated would improve productivity, quality as well as profitability.

Terminologies

Accuracy - The closeness of agreement between an observed value and the accepted reference value.

Appraiser Variation – It is the variation in average measurements of the same part (measurand) between different appraisers (operators) using the same measuring instrument and method in a stable environment.

Bias - The difference between the observed average of measurements (trials under repeatability conditions) and a reference value; historically referred to as accuracy. Bias is evaluated and expressed at a single point within the operating range of the measurement system.

Discrimination - Alias smallest readable unit, discrimination is the measurement resolution, scale limit, or smallest detectable unit of the measurement device and standard.

Linearity - The difference in bias errors over the expected operating range of the measurement system.

ndc - The number of data classifications or categories that can be reliably distinguished determined by the effective resolution of the measurement system and part variation from the observed process for a given application.

Part Variation - Related to measurement systems analysis, part variation (PV) represents the expected part-to-part and time-to-time variation for a stable process.

Repeatability - The common cause, random variation resulting from successive trials under defined conditions of measurement often referred to as equipment variation (EV), although this is misleading. The best term for repeatability is within-system variation when the conditions of measurement are fixed and defined – fixed part, instrument, standard, method, operator, environment, and assumptions. In addition to within equipment variation, repeatability will include all within variation from the conditions in the measurement error model.

Reproducibility - The variation in the average of measurements caused by a normal condition(s) of change in the measurement process. Typically, it has been defined as the variation in average measurements of the same part (measurand) between different appraisers (operators) using the same measuring instrument and method in a stable environment. Reproducibility is referred to as the average variation between-systems or between-conditions of measurement.

Resolution - The capability of the measurement system to detect and faithfully indicate even small changes of the measured characteristics.

Sensitivity - Smallest input signal that results in a detectable (discernable) output signal for a measurement device. An instrument should be at least as sensitive as its unit of discrimination.

Stability - Refers to both statistical stability of a measurement process and measurement stability over time. Statistical stability implies a predictable, underlying measurement process operating within common cause variation (in-control).

References

1. Daimler Chrysler Corporation, Ford Motor Company, General Motors Corporation, "Measurement Systems Analysis - Reference Manual", Third Edition, March 2002.